

Spin Hall effect in a system of Dirac fermions in the honeycomb lattice with intrinsic and Rashba spin-orbit interaction

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(Received 12 July 2009; revised manuscript received 21 September 2009; published 23 October 2009)

We consider spin Hall effect in a system of massless Dirac fermions in a graphene lattice. Two types of spin-orbit interaction, pertinent to the graphene lattice, are taken into account—the intrinsic and Rashba terms. Assuming perfect crystal lattice, we calculate the topological contribution to spin Hall conductivity. When both interactions are present, their interplay is shown to lead to some peculiarities in the dependence of spin Hall conductivity on the Fermi level.

DOI: [10.1103/PhysRevB.80.155444](https://doi.org/10.1103/PhysRevB.80.155444)

PACS number(s): 73.43.-f, 72.25.Hg, 73.61.Wp

I. INTRODUCTION

Long time ago Dyakonov and Perel predicted that in some systems transverse spin current and spin accumulation may be induced by electric current.^{1,2} This effect, known as spin Hall effect (SHE), is currently extensively studied both experimentally and theoretically^{3–8} (see also review papers^{9,10}). The effect may appear in semiconductors as well as in metals and originates from spin-orbit interaction. Generally, such an interaction may be of intrinsic or extrinsic origin. The extrinsic SHE is associated with scattering mechanisms, such as skew scattering and side jump in the presence of impurities. In turn, the intrinsic mechanism of SHE is a consequence of an unusual trajectory of the charge carriers in the momentum space, which may be described by the Berry phase formalism.^{11,12} This contribution will be referred to as the topological one.

In this paper we consider the topological contribution to spin Hall conductivity in a system of Dirac fermions with spin-orbit coupling. Generally, the form of spin-orbit interaction depends on the symmetry and structure of the system. The Dirac model turned out to be useful not only in the relativistic field theory but also in condensed-matter physics to describe some features of electronic spectrum (at least in a certain energy range). One of such systems is two-dimensional graphene and in this paper we consider Dirac fermions with the spin-orbit interaction taken in the form appropriate for graphene and including both intrinsic and Rashba terms. We believe that the results derived here will shed some light on the spin Hall effect in graphene.

Graphene is a two-dimensional honeycomb lattice of carbon atoms, with two nonequivalent sublattices. The low-energy electron states near the K and K' points at opposite corners of the Brillouine zone can be approximated by the conical energy spectrum. As a result, charge carriers are described by the Dirac equation.^{13,14} The Fermi surface in a neutral graphene consists of the nonequivalent points K and K', at which the valence and conduction bands touch each other. However, when the intrinsic spin-orbit interaction is

included, an energy gap opens at these points. Unfortunately, it is now believed that the intrinsic spin-orbit coupling in graphene is rather weak so the gap is also small. Since the graphene layer is usually on a substrate, one also can expect spin-orbit interaction of Rashba type.¹⁵ Moreover, the corresponding coupling parameter can be tuned externally by a gate voltage. Indeed, a large Rashba spin-orbit interaction has been reported in a recent experiment.¹⁶

Kane and Mele¹⁷ have shown that the intrinsic spin-orbit interaction opens an energy gap at the Dirac points and also have predicted a quantized value of the spin Hall conductivity when the Fermi level is in the gap. The quantized spin Hall conductivity at the Dirac points was also confirmed by later analytical and numerical calculations.^{18–20} On the other hand, the presence of Rashba spin-orbit interaction reduces the gap and when the Rashba interaction is stronger than the intrinsic one, the gap becomes closed. Since the magnitude of intrinsic spin-orbit interaction seems to be significantly smaller than that assumed originally,²¹ it is possible to reach the limit opposite to that considered by Kane and Mele, i.e., the limit where the Rashba coupling dominates while intrinsic spin-orbit interaction is negligible. To our knowledge, spin Hall effect in this limit has not been considered analytically so far. Therefore, in this paper we present analytical and numerical results obtained within the linear-response theory and Green's functions technique, assuming both intrinsic and Rashba spin-orbit interaction. We focus on the topological contribution to the effect assuming perfect crystal lattice and ignoring impurities and defects, which however may influence the magnitude of spin Hall effect in real systems.^{18–20,22}

The paper is organized as follows. In Sec. II we describe the model and present a general formula for the spin Hall conductivity. The case of pure intrinsic spin-orbit interaction is presented in Sec. III while the case of Rashba interaction is described and discussed in Sec. IV. The general case, where both interactions are present, is described in Sec. V while summary and final conclusions are in Sec. VI.

II. MODEL AND GENERAL FORMULA FOR SPIN HALL CONDUCTIVITY

Including both intrinsic and Rashba spin-orbit coupling, the effective-mass Hamiltonian¹⁷ of graphene can be written in the form

$$H = H_0 + H_{SO} + H_R. \quad (1)$$

The first term, H_0 , describes the low-energy electronic states around the Dirac points K and K' in the Brillouin zone and has the form

$$H_0 = \begin{pmatrix} 0 & v(\pm k_x - ik_y) \\ v(\pm k_x + ik_y) & 0 \end{pmatrix}, \quad (2)$$

where the upper and lower signs correspond to the points K and K' , respectively, and v is a parameter describing the conical energy spectrum, $v = \hbar v_F$, with v_F denoting the electron Fermi velocity. The second term in Eq. (1) describes the intrinsic spin-orbit interaction in graphene,

$$H_{SO} = \begin{pmatrix} \pm \Delta_{SO} \sigma_z & 0 \\ 0 & \mp \Delta_{SO} \sigma_z \end{pmatrix}, \quad (3)$$

with Δ_{SO} being the relevant parameter ($2\Delta_{SO}$ is the gap created by the intrinsic spin-orbit coupling in the Dirac points). Finally, the last term in Eq. (1) stands for the Rashba spin-orbit term,

$$H_R = \begin{pmatrix} 0 & \lambda_R(\pm \sigma_y + i\sigma_x) \\ \lambda_R(\pm \sigma_y - i\sigma_x) & 0 \end{pmatrix}, \quad (4)$$

where λ_R is the corresponding coupling parameter. As in Eq. (2), the upper and lower signs in Eqs. (3) and (4) correspond to the two inequivalent points K and K' of the Brillouin zone, respectively.

To obtain spin Hall conductivity we introduce first the spin current-density operator,

$$\mathbf{j}^{sj} = \frac{1}{2} [\mathbf{v}, s_j]_+, \quad (5)$$

where $[A, B]_+ = AB + BA$ denotes the anticommutator of any operators A and B , $v_i = (1/\hbar)(\partial H / \partial k_i)$ is the velocity operator ($i=x, y$) and $s_j = (\hbar/2)\sigma_j$ is the j th component ($j=x, y, z$) of the spin operator. Taking into account the exact form of Hamiltonian H one finds

$$v_x = \pm \frac{v}{\hbar} \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (6)$$

and

$$v_y = i \frac{v}{\hbar} \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad (7)$$

with I being the 2×2 unit matrix.

In the linear-response theory, the dc spin Hall conductivity is given by the formula,²³

$$\sigma_{xy}^{sz} = \lim_{\omega \rightarrow 0} \frac{e\hbar}{2\omega} \text{Tr} \int \frac{d\varepsilon}{2\pi} \frac{d^2\mathbf{k}}{(2\pi)^2} [v_x, s_z]_+ G_{\mathbf{k}}(\varepsilon + \omega) v_y G_{\mathbf{k}}(\varepsilon), \quad (8)$$

where $G_{\mathbf{k}}(\varepsilon)$ is the causal Green's function corresponding to Hamiltonian (1). This formula will be used in the following to calculate spin Hall conductivity in some specific cases as well as in a general situation.

III. CASE OF $\lambda_R=0$ AND $\Delta_{SO} \neq 0$

We will consider first the special case, when the Rashba coupling vanishes, while the intrinsic spin-orbit interaction is nonzero, $\lambda_R=0$ and $\Delta_{SO} \neq 0$. Such a situation has been already considered analytically in the clean limit¹⁷ and also studied numerically in the presence of impurities.¹⁸ Our results are consistent with those obtained in the above cited works.

When $\lambda_R=0$, Eq. (8) for the spin Hall conductivity takes the following form:

$$\sigma_{xy}^{sz} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d\varepsilon}{2\pi} \frac{i2ev^2\Delta_{SO}}{[\varepsilon - \sqrt{\Delta_{SO}^2 + v^2k^2} + \mu + i\delta \text{sign}(\varepsilon)]^2} \times \frac{1}{[\varepsilon + \sqrt{\Delta_{SO}^2 + v^2k^2} + \mu + i\delta \text{sign}(\varepsilon)]^2}, \quad (9)$$

where μ is the chemical potential.

In the zero-temperature limit, the spin Hall conductivity will be presented in the form

$$\sigma_{xy}^{sz} = \sigma_{xy}^{sz,0} \mp \delta\sigma_{xy}^{sz}, \quad (10)$$

where $\sigma_{xy}^{sz,0}$ is the contribution from the fully occupied valence band and $\delta\sigma_{xy}^{sz}$ is associated either with the empty part of the valence band (upper sign) or with occupied part of the conduction band (lower sign).

The contribution $\delta\sigma_{xy}^{sz}$ takes the form

$$\delta\sigma_{xy}^{sz} = \pm \frac{e\Delta_{SO}}{4\pi} \frac{1}{\sqrt{\Delta_{SO}^2 + v^2k^2}} \Big|_0^{k_F} = \frac{e}{4\pi} \left(1 \pm \frac{\Delta_{SO}}{\sqrt{\Delta_{SO}^2 + v^2k_F^2}} \right), \quad (11)$$

where the upper sign refers to the situation when the Fermi level is in the valence band while the lower one when the Fermi level is in the conduction band. In turn, the contribution from the fully occupied valence band can be calculated as

$$\sigma_{xy}^{sz,0} = - \frac{ev^2\Delta_{SO}}{4\pi} \int_0^\infty \frac{kdk}{(\Delta_{SO}^2 + v^2k^2)^{3/2}} = - \frac{e}{4\pi}. \quad (12)$$

Thus, from Eqs. (11) and (12) follows that the whole spin Hall conductivity can be written as

$$\sigma_{xy}^{sz} = - \frac{e}{4\pi} \frac{\Delta_{SO}}{|\mu|} \quad (13)$$

for $|\mu| > \Delta_{SO}$ and

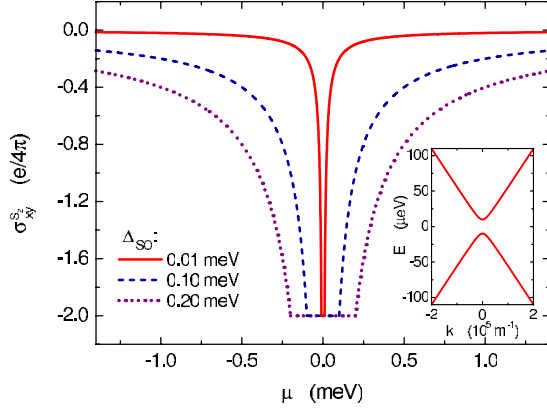


FIG. 1. (Color online) Spin Hall conductivity in the absence of Rashba interaction and for indicated values of the intrinsic spin-orbit parameter Δ_{SO} . The inset shows the energy spectrum in the vicinity of the Dirac point for $\Delta_{SO}=0.01$ meV. The parameter v is defined as $v=\hbar v_F$, where v_F is the Fermi velocity, $v_F=0.833 \times 10^6$ m/s.

$$\sigma_{xy}^{sz} = -\frac{e}{4\pi} \quad (14)$$

for $|\mu| < \Delta_{SO}$. The above result coincides with the ones obtained by Kane and Mele¹⁷ and Sinitsyn *et al.*¹⁹ The other two component of the spin Hall conductivity, i.e., σ_{xy}^{sx} and σ_{xy}^{sy} vanish, as one could expect.

The corresponding numerical results are shown in Fig. 1 for three different values of the parameter Δ_{SO} . Additional factor of 2 has been taken into account in order to include the two Dirac points K and K'. The spin Hall conductivity is shown there as a function of the chemical potential measured from the middle of the gap. When the Fermi level is in the energy gap, the Hall conductivity is constant and quantized. When, in turn, the Fermi level is either in the valence or conduction bands, the absolute value of conductivity decreases and disappears for $|\mu| \rightarrow \infty$. Note, the conductivity is symmetric with respect to the middle of the gap. The inset in Fig. 1 shows the energy spectrum in the vicinity of the Dirac point, calculated for the parameter v_F obtained by Gmitra *et al.*²⁴

IV. CASE OF $\Delta_{SO}=0$ AND $\lambda_R \neq 0$

Now, we consider the opposite situation, i.e., when the intrinsic spin-orbit coupling is negligible, $\Delta_{SO}=0$, while the Rashba parameter is nonzero, $\lambda_R \neq 0$. Equation (8) leads then to the following formula for the spin Hall conductivity:

$$\sigma_{xy}^{sz} = - \int \frac{d\varepsilon}{2\pi} \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{8iev^2\lambda_R^2(\varepsilon + \mu)[v^2(k_x^2 - k_y^2) + (\varepsilon + \mu)^2]}{\prod_{n=1}^4[\varepsilon - E_n(\mathbf{k}) + \mu + i\delta \text{sign}(\varepsilon)]} \times \frac{1}{\prod_{m=1}^4[\varepsilon - E_m(\mathbf{k}) + \mu + i\delta \text{sign}(\varepsilon)]}, \quad (15)$$

where $E_i(\mathbf{k})$ ($i=1-4$) describe the electron energy spectrum,

$$E_1(\mathbf{k}) = \lambda_R + (\lambda_R^2 + v^2k^2)^{1/2}, \quad (16)$$

$$E_2(\mathbf{k}) = \lambda_R - (\lambda_R^2 + v^2k^2)^{1/2}, \quad (17)$$

$$E_3(\mathbf{k}) = -\lambda_R + (\lambda_R^2 + v^2k^2)^{1/2}, \quad (18)$$

$$E_4(\mathbf{k}) = -\lambda_R - (\lambda_R^2 + v^2k^2)^{1/2}. \quad (19)$$

The states $E_1(\mathbf{k})$ and $E_3(\mathbf{k})$ correspond to the conduction bands while $E_2(\mathbf{k})$ and $E_4(\mathbf{k})$ to the valence bands. We consider first the case when $|\mu| > 2\lambda_R$.

A. Case of $|\mu| > 2\lambda_R$

When the Fermi level is in the two valence bands, $\mu < -2\lambda_R$, then upon integrating Eq. (15) over ε one arrives at the following formula:

$$\sigma_{xy}^{sz} = \frac{ev^2}{16\pi\lambda_R} \int dk \frac{2\lambda_R^2k + v^2k^3}{(v^2k^2 + \lambda_R^2)^{3/2}} [f(\varepsilon_2) - f(\varepsilon_4)], \quad (20)$$

where $f(\varepsilon)$ is the Fermi-Dirac distribution (assumed here for $T=0$). Taking now into account the notation introduced in Eq. (10), one finds

$$\sigma_{xy}^{sz,0} = 0 \quad (21)$$

and

$$\delta\sigma_{xy}^{sz} = -\frac{e}{4\pi} \frac{\mu^2}{2(\mu^2 - \lambda_R^2)}. \quad (22)$$

As before, $\sigma_{xy}^{sz,0}$ is the contribution from both fully occupied valence bands while $\delta\sigma_{xy}^{sz}$ takes into account the empty part of the valence bands. Now, the contributions from the two fully occupied valence bands cancel each other, so $\sigma_{xy}^{sz,0}$ vanishes exactly.

When the Fermi level is in both conduction bands, $\mu > 2\lambda_R$, the spin Hall conductivity can be calculated in a similar way and the formula for $\delta\sigma_{xy}^{sz}$ takes the same form as for $\mu < 2\lambda_R$, i.e., Eq. (22). Thus, the total spin Hall conductivity for $|\mu| > 2\lambda_R$ can be written as

$$\sigma_{xy}^{sz} = \frac{\mu^2}{2(\mu^2 - \lambda_R^2)} \frac{e}{4\pi} \quad (23)$$

for the Fermi level in conduction bands, $\mu > 2\lambda_R$, and

$$\sigma_{xy}^{sz} = -\frac{\mu^2}{2(\mu^2 - \lambda_R^2)} \frac{e}{4\pi} \quad (24)$$

for the Fermi level in the valence bands, $\mu < 2\lambda_R$.

B. Case of $|\mu| < 2\lambda_R$

Now, we assume that $|\mu| < 2\lambda_R$. The only difference is that now the Fermi level is either in one valence band or in one conduction band. Spin Hall conductivity can be calculated in a similar way as before and one finds

$$\sigma_{xy}^{sz} = \frac{\mu(\mu + 2\lambda_R)}{4\lambda_R(\mu + \lambda_R)} \frac{e}{4\pi} \quad (25)$$

for the Fermi level in the conduction band and

$$\sigma_{xy}^{sz} = \frac{\mu(\mu - 2\lambda_R)}{4\lambda_R(\mu - \lambda_R)} \frac{e}{4\pi} \quad (26)$$

for the Fermi level in the valence band.

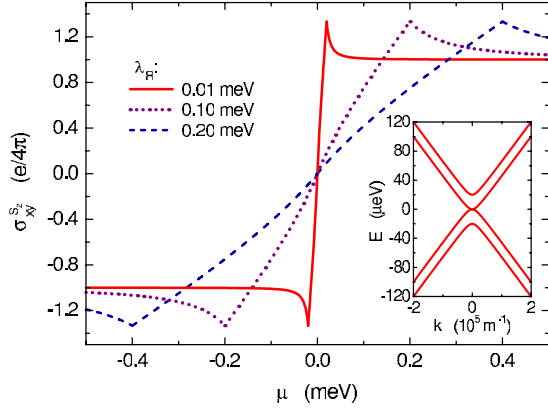


FIG. 2. (Color online) Spin Hall conductivity in the absence of intrinsic spin-orbit interaction and for indicated values of the Rashba coupling parameter λ_R . The other parameters are as in Fig. 1. The inset shows the energy spectrum in the vicinity of the energy gap for $\lambda_R = 0.01$ meV.

Figure 2 presents numerical results for the spin Hall conductivity as a function of the chemical potential. As before, we included a factor of 2 in order to take into account contribution from the second Dirac point. We note that now the spin Hall conductivity tends to $\sigma_{xy}^{s_z} = -e/4\pi$ in the limit of $\mu \rightarrow -\infty$ while for $\mu \rightarrow \infty$ it tends to the $\sigma_{xy}^{s_z} = e/4\pi$. This behavior is different from that for intrinsic spin-orbit interaction, where the contribution from conduction bands canceled the corresponding part from the valence bands. It is also worth to note that now the spin Hall conductivity is antisymmetric with respect to change in the Fermi level sign.

We have also checked the other components of the spin Hall conductivity. As before, these components vanish exactly, $\sigma_{xy}^{s_x} = \sigma_{xy}^{s_y} = 0$.

V. CASE WITH $\lambda_R \neq 0$ AND $\Delta_{SO} \neq 0$

When both intrinsic and Rashba spin-orbit interactions are present in the system, their interplay leads to interesting and peculiar behavior of the spin Hall conductivity. The analytical formulas, however, are much too complex to be presented here, so we will show mainly results of numerical calculations.

Writing the Green's function as

$$G_{\mathbf{k}}(\varepsilon) = \frac{g_{\mathbf{k}}(\varepsilon)}{\prod_{n=1}^4 [\varepsilon - E_n(\mathbf{k}) + \mu + i\delta \text{sign}(\varepsilon)]}, \quad (27)$$

where $E_n(\mathbf{k})$ ($n=1-4$) are the dispersion relations for the conduction and valence bands,

$$E_1(\mathbf{k}) = \lambda_R + \sqrt{(\Delta_{SO} - \lambda_R)^2 + v^2 k^2}, \quad (28)$$

$$E_2(\mathbf{k}) = \lambda_R - \sqrt{(\Delta_{SO} - \lambda_R)^2 + v^2 k^2}, \quad (29)$$

$$E_3(\mathbf{k}) = -\lambda_R + \sqrt{(\Delta_{SO} + \lambda_R)^2 + v^2 k^2}, \quad (30)$$

$$E_4(\mathbf{k}) = -\lambda_R - \sqrt{(\Delta_{SO} + \lambda_R)^2 + v^2 k^2}, \quad (31)$$

and taking into account the explicit form of v_x and v_y one may write the relevant trace in Eq. (8) as

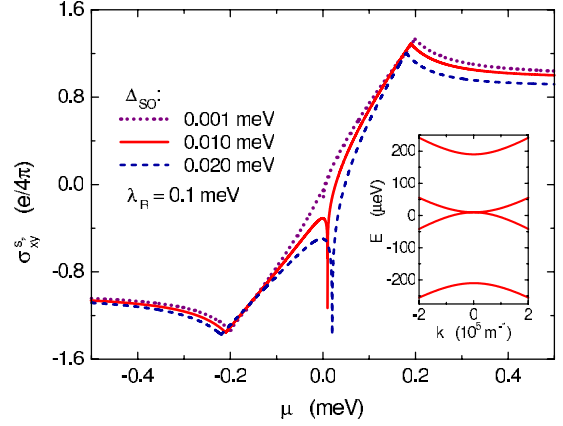


FIG. 3. (Color online) Spin Hall conductivity for $\lambda_R = 0.1$ meV and indicated values of Δ_{SO} , $\Delta_{SO} < \lambda_R$. The inset shows the energy spectrum in the vicinity of the Dirac point for $\Delta_{SO} = 0.01$ meV. The parameter v is taken as in Fig. 1.

$$\text{Tr} \left\{ \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix} g_{\mathbf{k}}(\varepsilon_1) \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} g_{\mathbf{k}}(\varepsilon_2) \right\} = \mathcal{P}(\varepsilon_1, \varepsilon_2), \quad (32)$$

where $\mathcal{P}(\varepsilon_1, \varepsilon_2)$ is a certain function of $\varepsilon_1 = \varepsilon + \mu + \omega$ and $\varepsilon_2 = \varepsilon + \mu$. Expanding $\mathcal{P}(\varepsilon_1, \varepsilon_2)$ as

$$\mathcal{P}(\varepsilon_1, \varepsilon_2) = \mathcal{P}(\varepsilon_1, \varepsilon_2)|_{\varepsilon_1=\varepsilon_2=\varepsilon+\mu} + \omega \left. \frac{\partial \mathcal{P}(\varepsilon_1, \varepsilon_2)}{\partial \varepsilon_1} \right|_{\varepsilon_1=\varepsilon_2=\varepsilon+\mu} + \dots \quad (33)$$

and taking into account that $\mathcal{P}(\varepsilon_1, \varepsilon_2)|_{\varepsilon_1=\varepsilon_2=\varepsilon+\mu} = 0$, one finds

$$\begin{aligned} \mathcal{P}(\varepsilon + \omega, \varepsilon) \cong & -\omega \{ -4[(k^2 v^2 + \Delta_{SO}^2)^2 + 4v^2 \lambda_R^2 (k_x^2 - k_y^2)] \\ & + 16\lambda_R^2 [\Delta_{SO}^2 + v^2 (k_x^2 - k_y^2)] (\varepsilon + \mu) \\ & + 8\Delta_{SO} (v^2 k^2 + \Delta_{SO}^2 - 4\lambda_R^2) (\varepsilon + \mu)^2 \\ & + 16\lambda_R^2 (\varepsilon + \mu)^3 - 4\Delta_{SO} (\varepsilon + \mu)^4 \}. \end{aligned} \quad (34)$$

These formulas can be then used to calculate spin Hall conductivity, either analytically by integrating over ε and \mathbf{k} or numerically. Since analytical formula is generally rather cumbersome, we performed numerical calculations while analytical formula will be presented only for some special cases (see below).

A. $\lambda_R > \Delta_{SO}$

Consider first the case when λ_R is significantly larger than Δ_{SO} . The corresponding spin Hall conductivity is shown in Fig. 3 for $\lambda_R = 0.1$ meV and three different values of Δ_{SO} (smaller than λ_R). When Δ_{SO} is much smaller than λ_R , then the conductivity is determined practically only by the Rashba coupling. With increasing Δ_{SO} , the interplay of both interactions leads to anomalous behavior of the spin Hall conductivity. More specifically, the conductivity becomes diverging when the Fermi level approaches the point, at which the top of the upper valence band touches the bottom of the lower conduction band (see the inset in Fig. 3). This appears when $\mu = \Delta_{SO}$. Asymptotic behavior of the conduc-

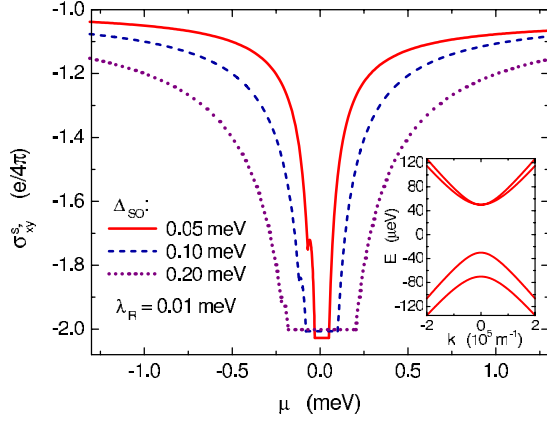


FIG. 4. (Color online) Spin Hall conductivity for $\lambda_R=0.01$ meV and indicated values of Δ_{SO} , $\Delta_{SO} > \lambda_R$. The inset shows the energy spectrum in the vicinity of the Dirac point for $\Delta_{SO}=0.05$ meV. The parameter v is taken as in Fig. 1.

tance near the point $\mu=\Delta_{SO}$ is described by the term $(\Delta_{SO}/2\lambda_R)\ln[-\Delta_{SO}-\lambda_R+\sqrt{(\lambda_R+\mu)^2}]$ when $\mu=\Delta_{SO}$ is approached from the right ($\mu > \Delta_{SO}$) side and $(\Delta_{SO}/2\lambda_R)\ln[\Delta_{SO}-\lambda_R+\sqrt{(\lambda_R-\mu)^2}]$ when it is approached from the left ($\mu < \Delta_{SO}$) side.

B. $\lambda_R < \Delta_{SO}$

Let us now consider the opposite situation, when λ_R is smaller than Δ_{SO} . The corresponding spin Hall conductivity is shown in Fig. 4 for $\lambda_R=0.01$ meV and three different values of Δ_{SO} (larger than λ_R). General shape of the curve showing spin Hall conductivity as a function of the chemical potential is similar to that for $\lambda_R=0$. However, the interplay of intrinsic spin-orbit interaction and Rashba coupling leads to an interesting feature. More specifically, there is now no divergence but a weak kink in the conductance appears on the negative chemical-potential side. When Δ_{SO} decreases and approaches λ_R , the kink becomes more pronounced. The kink is associated with splitting of the valence band by the Rashba interaction. The upper valence-band edges are now at $-\Delta_{SO}-2\lambda_R$ and $-\Delta_{SO}+2\lambda_R$.

C. $\lambda_R = \Delta_{SO}$

Variation in the spin Hall conductivity with the Fermi level becomes more complex when both Δ_{SO} and λ_R are comparable. Some simple analytical results, however, can be obtained for $\lambda_R=\Delta_{SO}$. For this particular case, the bottom edges of two conduction bands coincide with top edge of one of the valence band while the top edge of the second valence band is much below (see the inset in Fig. 5).

The relevant formula for the spin Hall conductivity depends then on the Fermi level as follows:

1. $\mu < -3\Delta_{SO}$

For $\mu < -3\Delta_{SO}$, the spin Hall conductivity is given by the formula

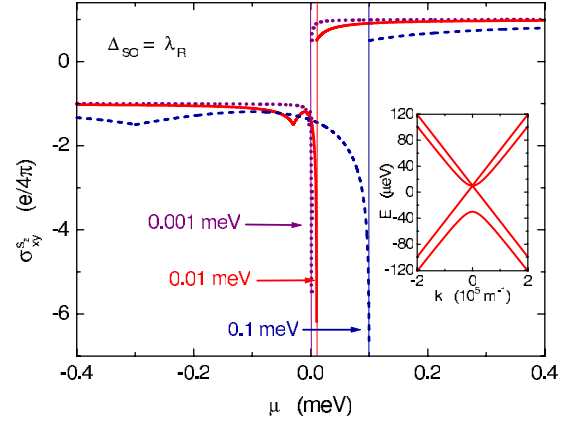


FIG. 5. (Color online) Spin Hall conductivity for indicated values of $\lambda_R=\Delta_{SO}$. The inset shows the energy spectrum in the vicinity of the Dirac point for $\lambda_R=\Delta_{SO}=0.01$ meV. The parameter v is taken as in Fig. 1. The vertical dotted lines indicate the position where the anomaly appears.

$$\sigma_{xy}^{S_z} = -\frac{\mu}{\mu + \Delta_{SO}} \frac{e}{4\pi}. \quad (35)$$

This formula covers the energy range up to the top edge of the lower valence band.

2. $-3\Delta_{SO} < \mu < \Delta_{SO}$

When $-3\Delta_{SO} < \mu < \Delta_{SO}$, the corresponding formula for the conductivity takes the form

$$\sigma_{xy}^{S_z} = \left[\frac{\mu}{2\Delta_{SO}} + \ln\left(\frac{\Delta_{SO}-\mu}{4\Delta_{SO}}\right) \right] \frac{e}{4\pi}. \quad (36)$$

This formula, in turn, describes spin Hall conductivity when the chemical potential is between the top edge of the lower valence band and bottom edges of the conduction bands. Note that this formula leads to diverging spin Hall conductivity when the Fermi level tends from left (lower values) to Δ_{SO} .

3. $\mu > \Delta_{SO}$

Finally, for $\mu > \Delta_{SO}$ the spin Hall conductivity is equal to

$$\sigma_{xy}^{S_z} = \frac{\mu}{\mu + \Delta_{SO}} \frac{e}{4\pi}. \quad (37)$$

This formula gives a finite spin Hall conductivity in the whole range of its applicability, also at the point $\mu=\Delta_{SO}$.

The spin Hall conductivity for $\lambda_R=\Delta_{SO}$ is shown in Fig. 5 for indicated values of $\lambda_R=\Delta_{SO}$. The anomaly at $\mu=\lambda_R=\Delta_{SO}$ is now clearly visible. When the chemical potential μ tends to $\mu=\lambda_R=\Delta_{SO}$ from the right ($\mu > \Delta_{SO}$) side, the conductivity is finite while when it tends to $\mu=\lambda_R=\Delta_{SO}$ from the left ($\mu < \Delta_{SO}$) side, the conductivity becomes diverging. The vertical dotted lines in Fig. 5 indicate only the position where the anomaly appears.

VI. SUMMARY AND DISCUSSION

Assuming intrinsic and Rashba spin-orbit interaction we have calculated topological contribution to the spin Hall con-

ductivity. In the limit of vanishing Rashba term we arrived at the results which are in agreement with those available in the relevant literature. When, in turn, the Rashba coupling dominates and intrinsic spin-orbit coupling vanishes, we have found asymmetric behavior of the spin Hall conductivity with respect to the sign reversal of the chemical potential. Such a change in the chemical potential can be achieved with a gate voltage, for instance.

When both intrinsic and Rashba spin-orbit interactions are present, their interplay leads to some peculiar and anomalous behavior of the spin hall conductivity with the Fermi level.

In particular, for some range of spin-orbit parameters, the spin Hall conductivity was found to diverge when the Fermi level approaches the limit $\mu = \Delta_{SO}$.

ACKNOWLEDGMENTS

This work was supported by the EU grant CARDEQ under Contract No. IST-021285-2, FCT under Grant No. PTDC/FIS/70843/2006 in Portugal, and by funds from the Ministry of Science and Higher Education as a research project in years 2007–2010.

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